**Probability distributions**

**(theoretical properties and characteristics + generation of random numbers + sample characteristics)**

**Evaluation: 0-5 points**

**Deadline: see InSIS**

You should use a Coursework submissions application in InSIS. Apply R for computations. Scans of handwriting or word (Latex - pdf) documents with comments could be added. In case you used different software for calculations, save the results in PDF formatting.

# Example 1

You will work with lognormal distribution. Select a pair of parameters  and for selected distribution:

* Draw the density, the CDF and the quantile function.
* Evaluate median, quartiles, expected value, mode, variance and standard deviation.
* Use known formulas and implemented functions (in R):
* Generate 50, 100 and 200 independent random numbers from selected distributions (hence 3 samples). For every sample:
  + Find the mean, median, sample (!) variance, sample standard deviation and sample quartiles.
  + Construct a table of theoretical and sample values (sample counterparts) in order to illustrate relationship between theoretical and sample characteristics (they should be “comparable” but not equivalent).
* Draw histograms and add the theoretical (log-normal) density function into them.

# Example 2

Beta distribution is very flexible in the shape of its density function, we can model well very different distributions. Select 5 pairs of parameters:

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Draw densities and cumulative distribution functions of these distributions. Evaluate the expected values, medians and modes (or antimodes).

# Example 3

Here gamma distribution is used. Select an arbitrary pair of parameters (*k*> 1) of the distribution and for *n* = 25, 50, and 100 use a property of gamma distribution (convolution in Lecture 1) to find a distribution (name, parameters) for the total *M*  and evaluate:

* the expected value, median, mode, lower and upper quartiles, variance, standard deviation, quartile deviation and the coefficient of skewness.
* Select (small) and evaluate 
* Use the central limit theorem to approximate the distribution of *M* (for *n*= 50 and 100). Evaluate same characteristics and probabilities as in previous two points.
* Compare results in a table.

# Example 4

Suppose random sample of sample size 1,000.

* Find out (using Chebyshev inequality and then CLT) the probability, that the relative frequency *P* of faulty products in the sample will diverge from its expected value 0.03 by 0.01 at max.
* Use Chebyshev inequality and CLT to find out the sample sizes (n1 andn2) needed for this to hold true:
* Find out the sample size (n3) needed for the standard deviation of relative frequency of faulty products to by 0.01 at max.
* Find out the sample size (n4) needed for the sample to contain at least 1,000 non-faulty products with the probability at least 0.9.
* Find out the sample size (n5) needed for the expected value of non-faulty products to be at least 1,000.